

What do you do if your garden hose does not reach all of your plants?



What does this do?
Why?



Main Ideas in Class Today

After today's class, you should be able to:

- Understand how liquid/gas **velocity** and **pressure** changes if you change the area of a pipe (utilize equation of continuity)
- Apply the **Bernoulli** principle to airplane flight, your arteries and hurricane winds

Suggested Practice Problems: C9.3, C9.7, C9.13, 9.17, 9.21, 9.23, 9.25, 9.27, 9.31

May have time at class end for movie review questions. If not, office hours or GroupMe

What is the continuity equation?

Due to the conservation of mass.

Unless the hose/pipe can expand, the same amount of water going into the hose comes out.



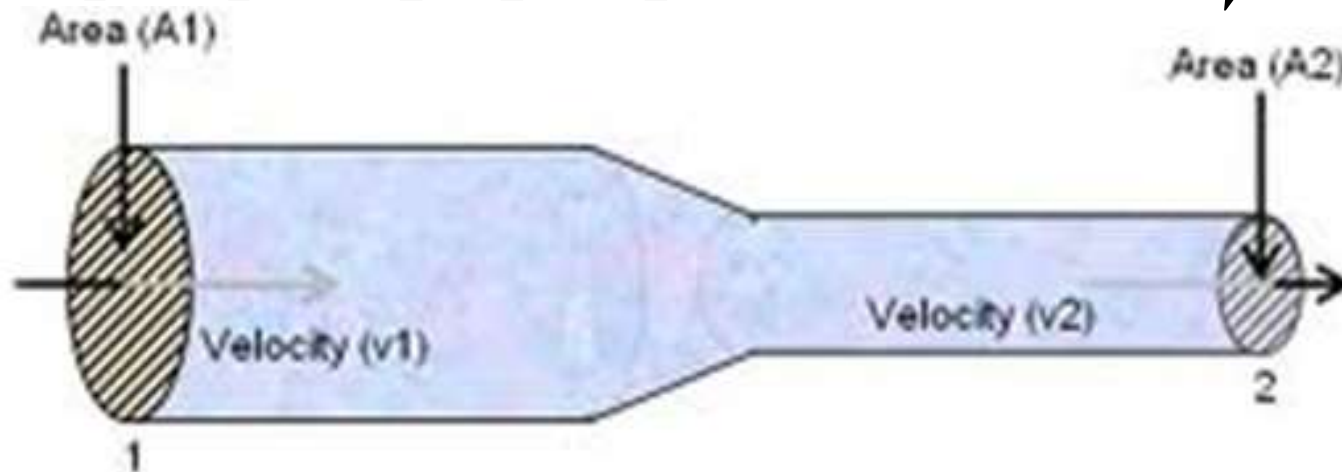
What happens if the size of the hose changes?

Deriving the Continuity Equation

- Mass 1 = Mass 2 (actually rate, but add later)
- Like in the last class, we can use $M = \rho \text{ Vol}$
- $\rho_1 \text{ Vol}_1 = \rho_2 \text{ Vol}_2$
- Like last time, Volume equals cross sectional area A times some depth Δx

- $\rho_1 A_1 \Delta x_1 = \rho_2 A_2 \Delta x_2$

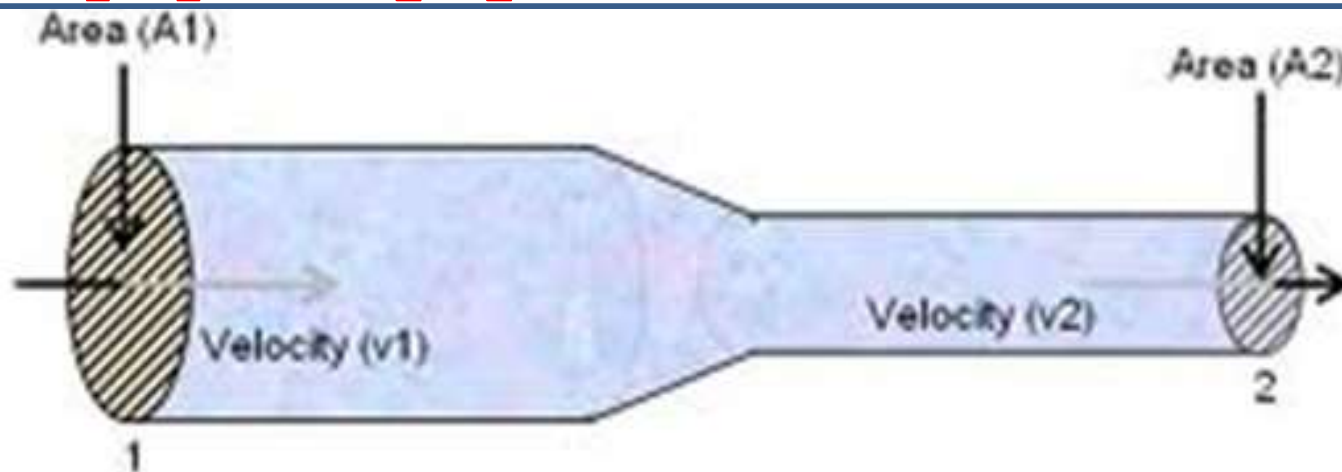
$$\rho = \frac{M}{V} = \frac{\text{mass}}{\text{volume}}$$



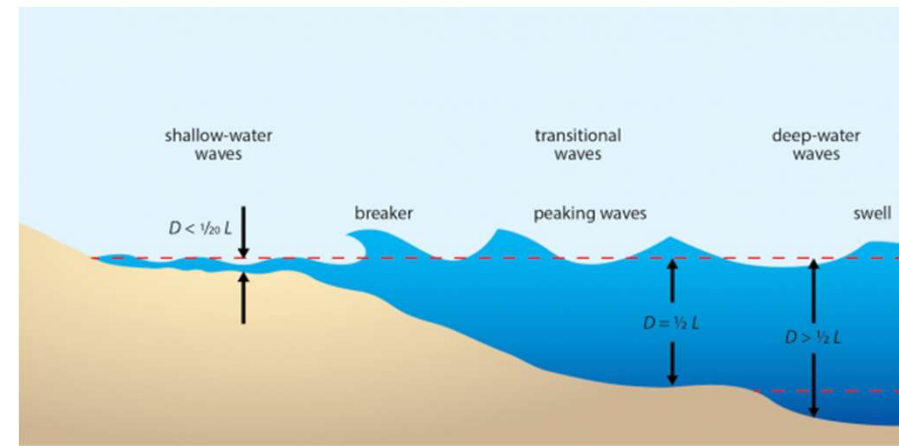
Deriving the Continuity Equation

- Normally we are talking about one liquid, so $\rho_1 = \rho_2$, so this simplifies to $A_1 \Delta x_1 = A_2 \Delta x_2$
- Instead of mass, we discuss the mass rate, or M/time or $A_1 \Delta x_1 / t = A_2 \Delta x_2 / t$
- What is $\Delta x / t$?

$$A_1 v_1 = A_2 v_2 \text{ (equation of continuity)}$$



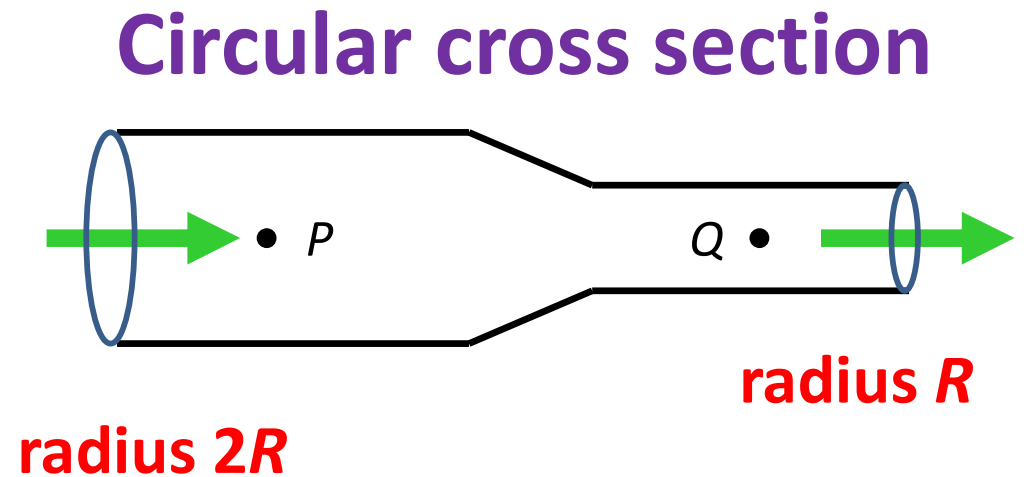
Equation of Continuity Implications



- The product, Av , is called the *volume flux* or the *flow rate*
 - The speed is high where the tube is constricted (small A)
 - The speed is low where the tube is wide (large A)
- True if no leaks are present



An fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point P , the fluid at point Q has



- A. 4 times the fluid speed.
- B. 2 times the fluid speed.
- C. the same fluid speed.
- D. $1/2$ the fluid speed.
- E. $1/4$ the fluid speed.

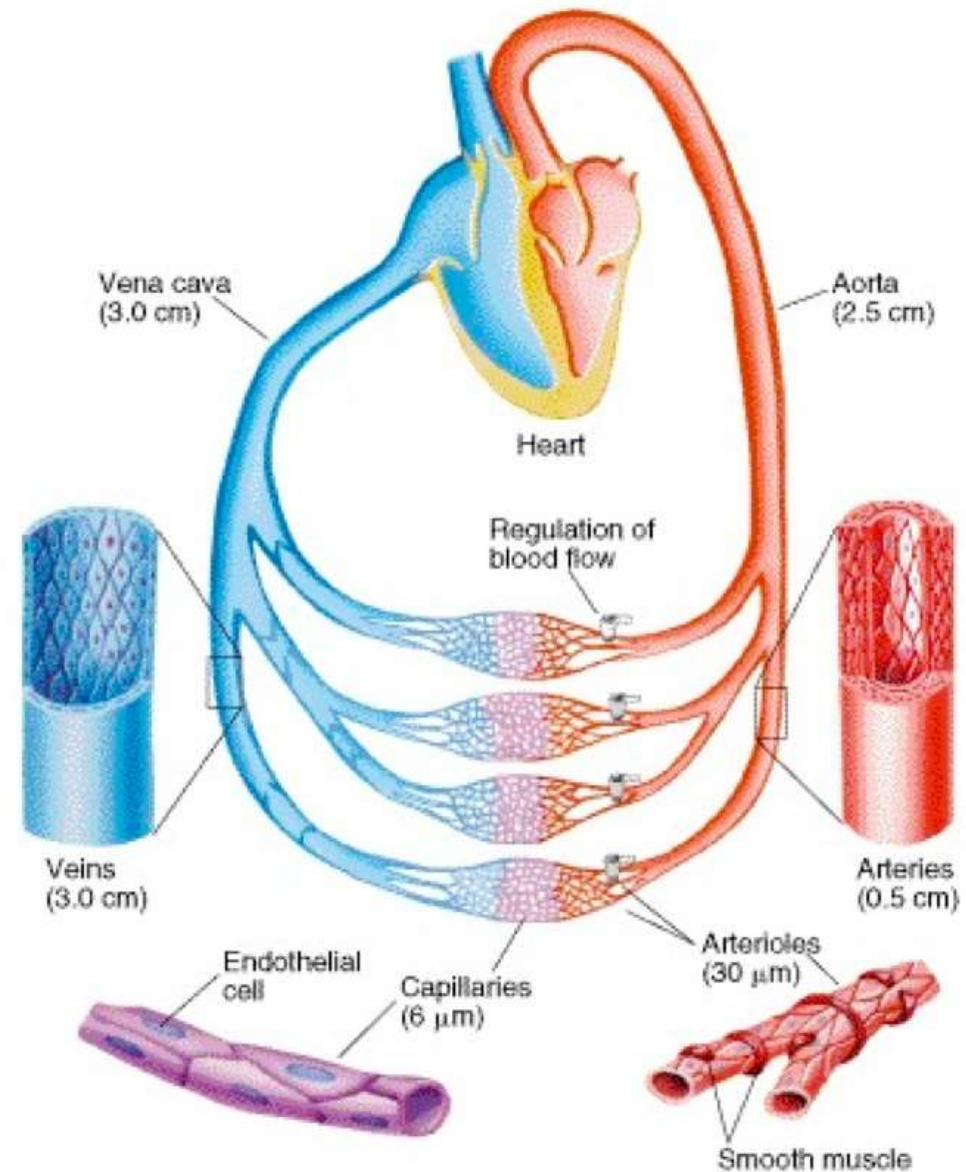


Q125

Veins are like pipes

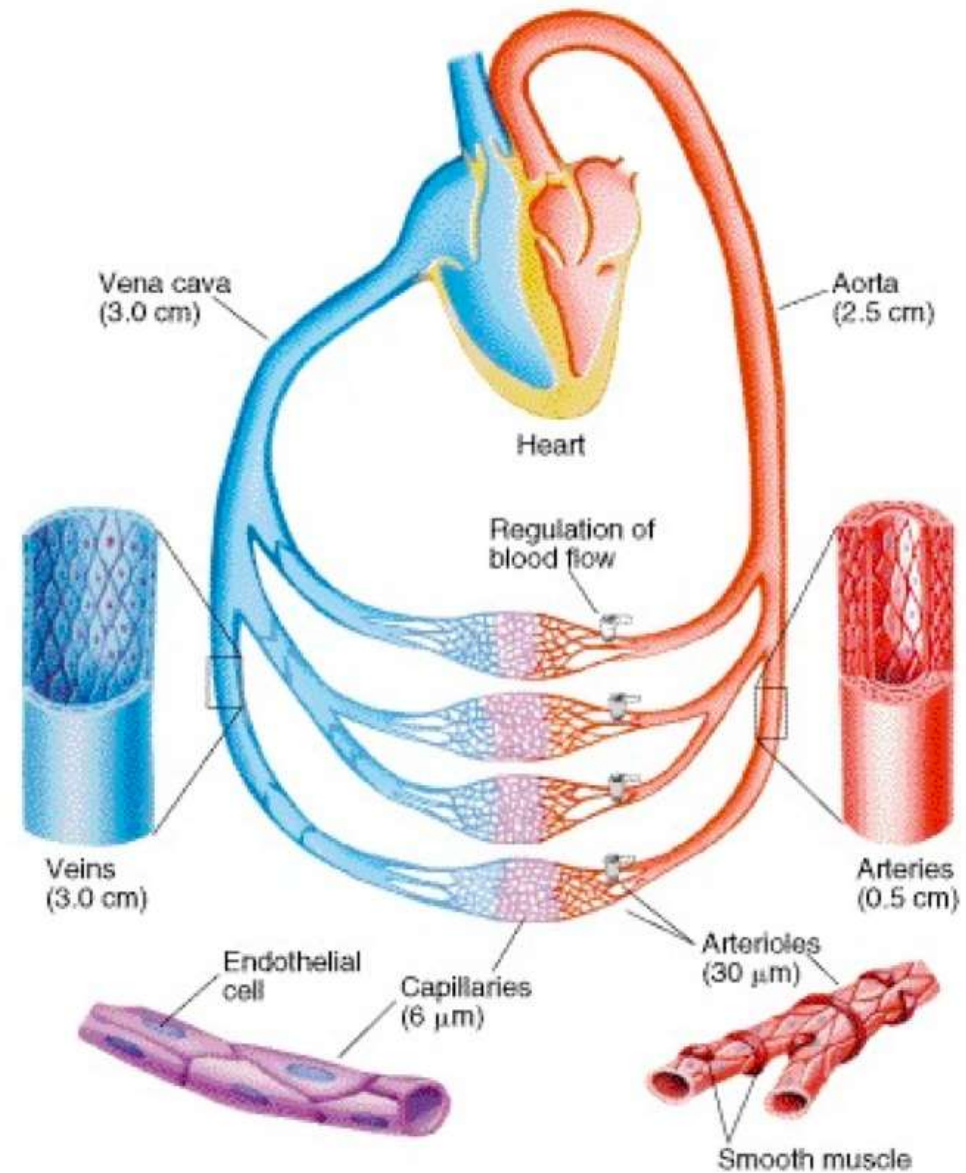
We are going to use this principle to estimate the number of capillaries in the circulatory system.

Chat with your neighbor about the kinds of information you might need to make this estimation.



Veins are like pipes

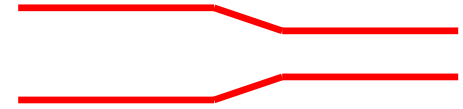
The approximate inside diameter of the aorta is 2.5 cm; that of a capillary is $6\ \mu\text{m}$. The approximate average blood flow speed is 1.0 m/s in the aorta and 1.0 cm/s in the capillaries. If all the blood in the aorta eventually flows through the capillaries, estimate the number of capillaries in the circulatory system.



Bernoulli's Equation



Not all pipes are horizontal.

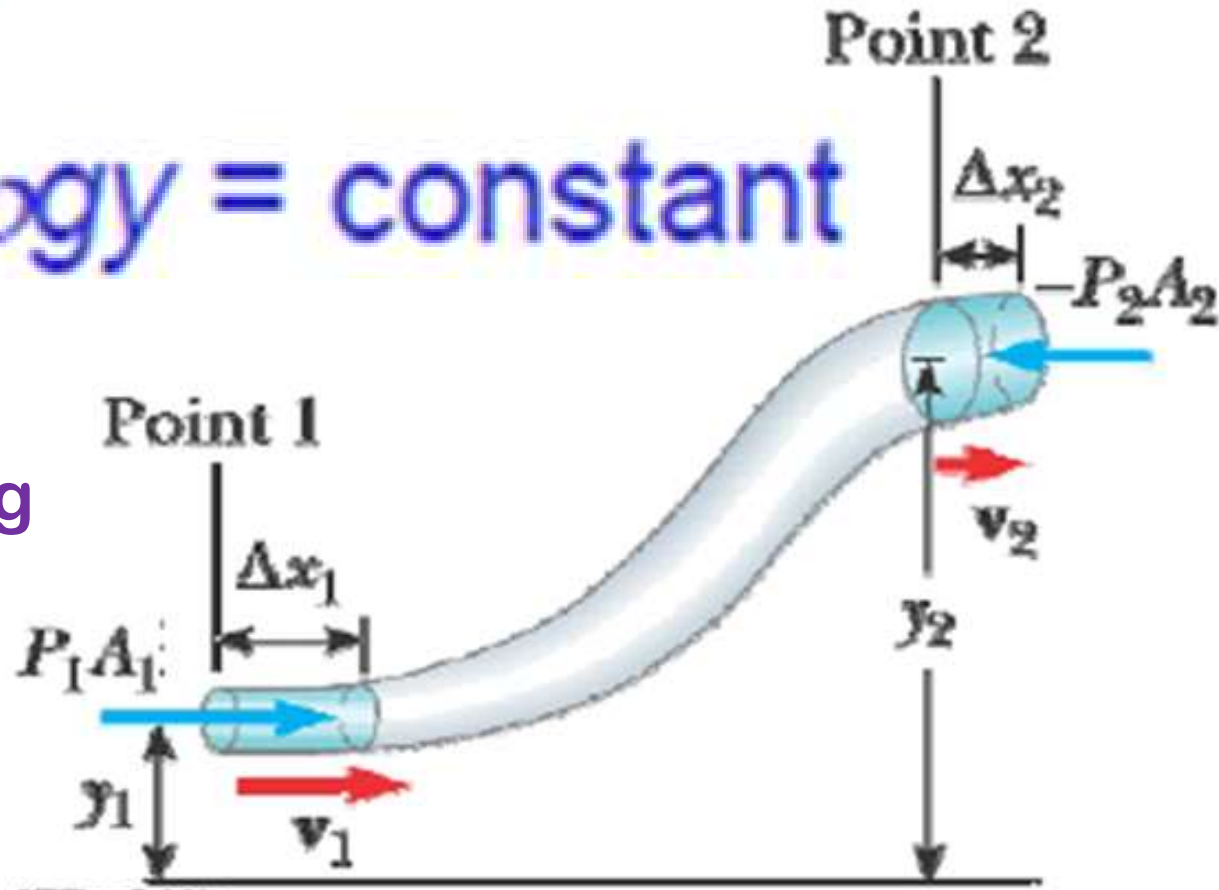


- As a fluid moves through a region where its speed and/or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes

$$P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}$$



Book derives this by using conservation of energy and $Av = \text{constant}$.

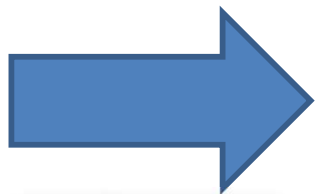
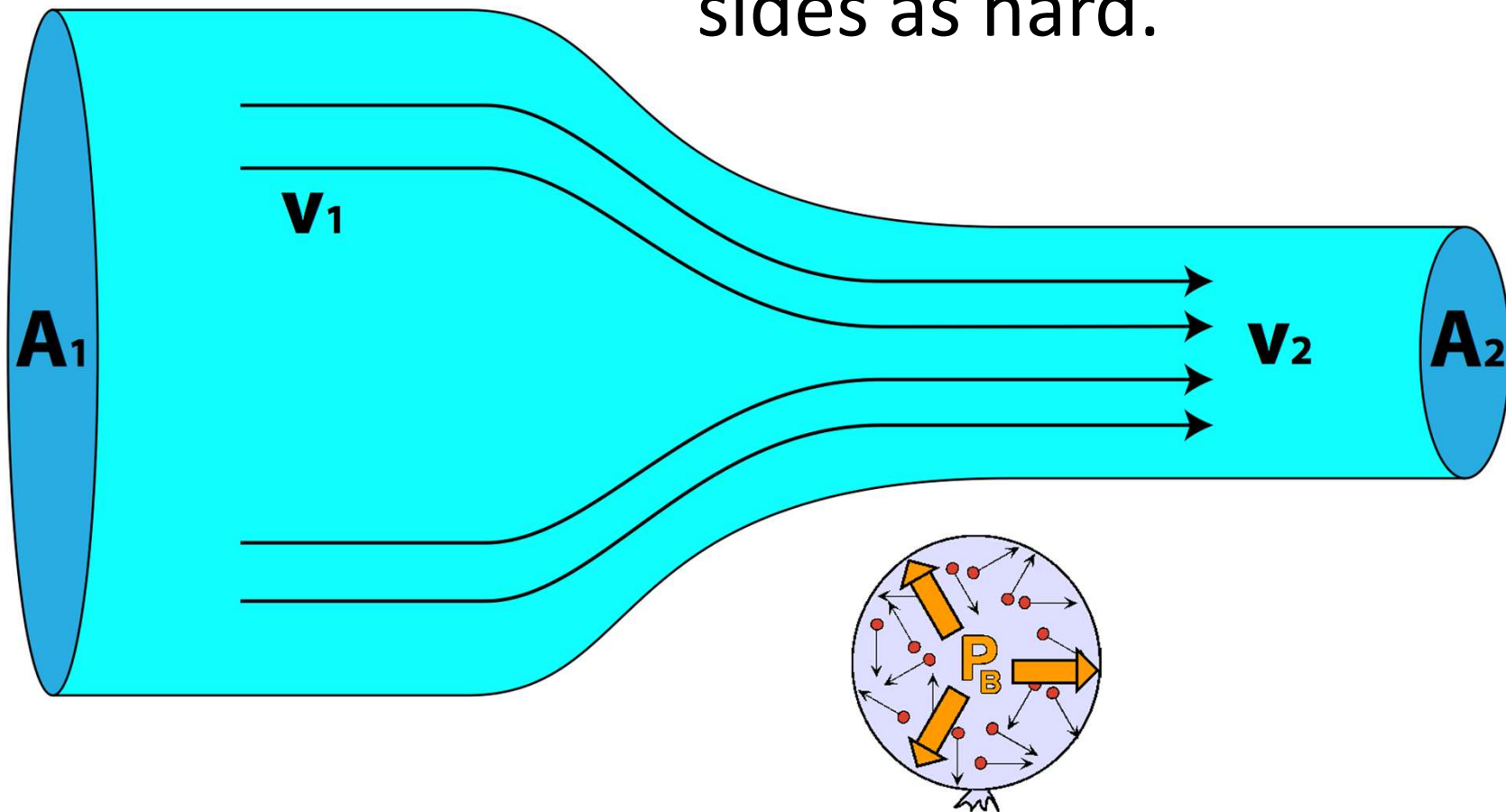


- When the fluid is at rest, this becomes $P_1 - P_2 = \rho g h$ which is consistent with the pressure variation with depth we found earlier
- The general behavior of pressure with speed is true even for gases
 - As the speed increases, the pressure decreases **Counter-intuitive!**

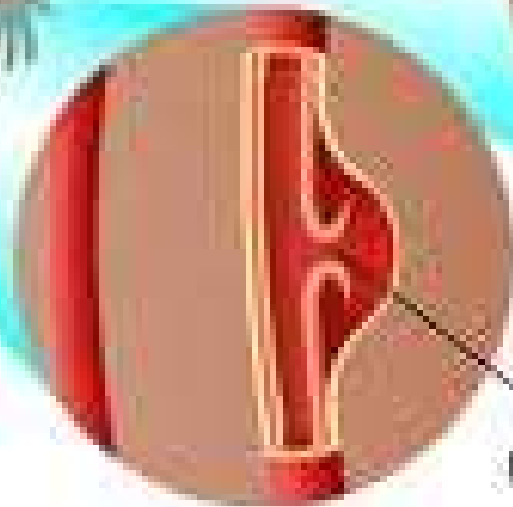
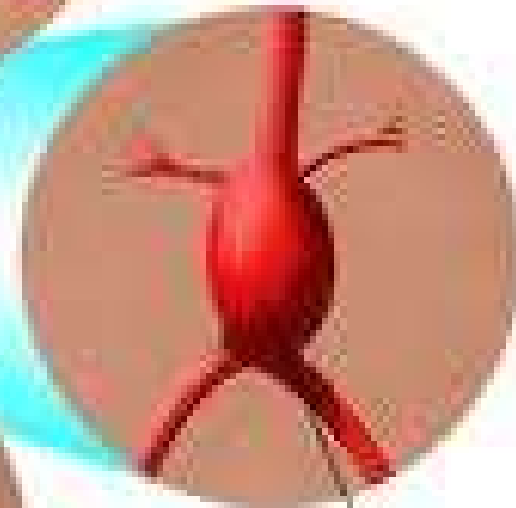
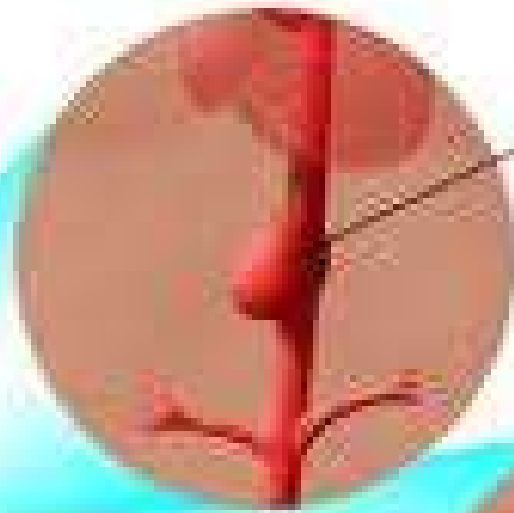
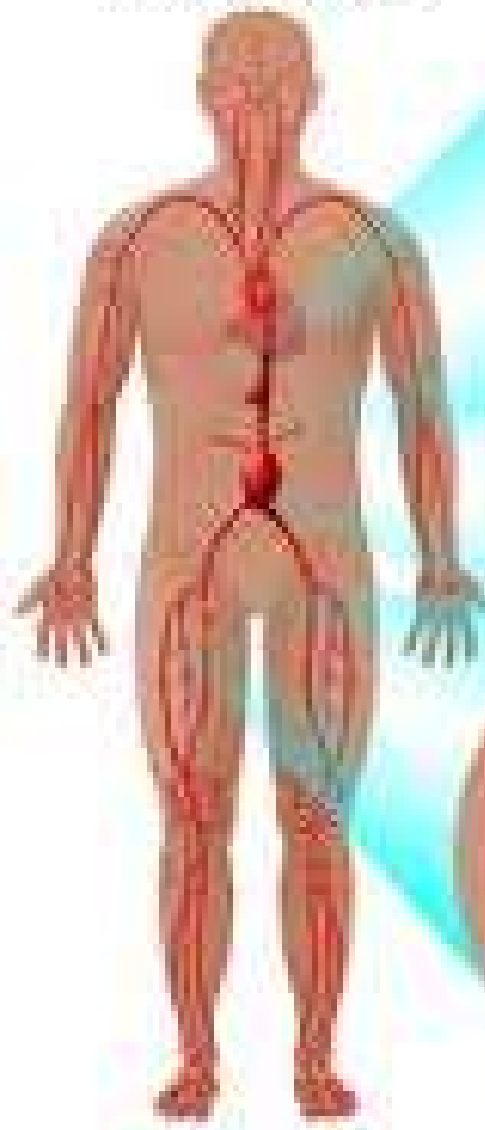
Why does this seem confusing?

By pressure here we mean the pressure on the sides of the pipes. Not the pressure of liquid coming out.

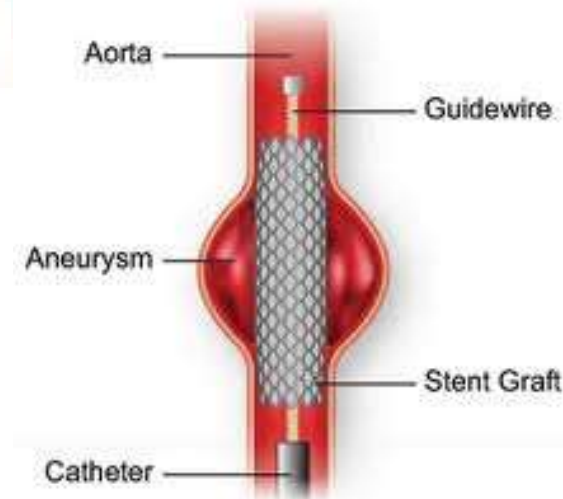
Since the liquid goes faster, it can't push on the sides as hard.



Types of Aneurysms

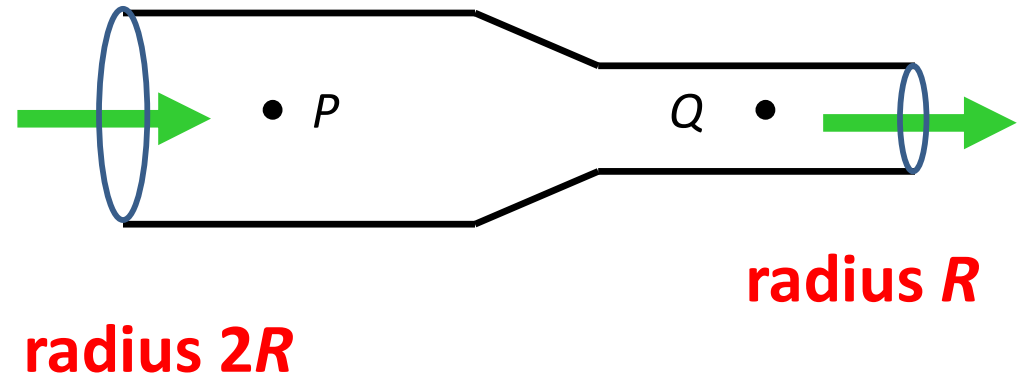


Ballooning causes increased area and decreased velocity, therefore increased pressure!



Increased pressure can cause the artery to rupture

An incompressible fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point P , the fluid at point Q has



$$\text{volume flow rate} = Av$$

- A. greater pressure and greater volume flow rate.
- B. greater pressure and the same volume flow rate.
- C. the same pressure and greater volume flow rate.
- D. lower pressure and the same volume flow rate.
- E. none of the above

$$P + \frac{1}{2} \rho v^2 + \rho gy = \text{constant}$$



Q126

Consider a house with a thin flat roof of area 1600 square feet (148.6 square meters). During a hurricane with winds of 140 mph, what is the net force on the roof?

Outside house



2

1

$v_{\text{air}} \sim 0$

Inside house

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

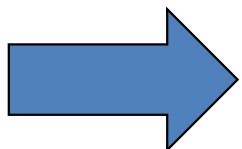
$$y_2 \approx y_1$$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

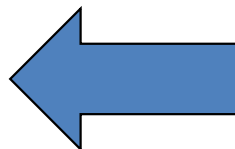
$$\Delta P = P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) = F / A$$

If the wind is blowing horizontally very hard outside, what direction does the force on the roof due to the change in air pressure point?

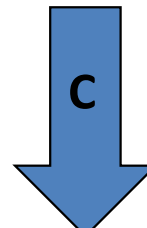
A



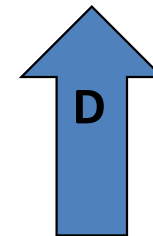
B



C



D



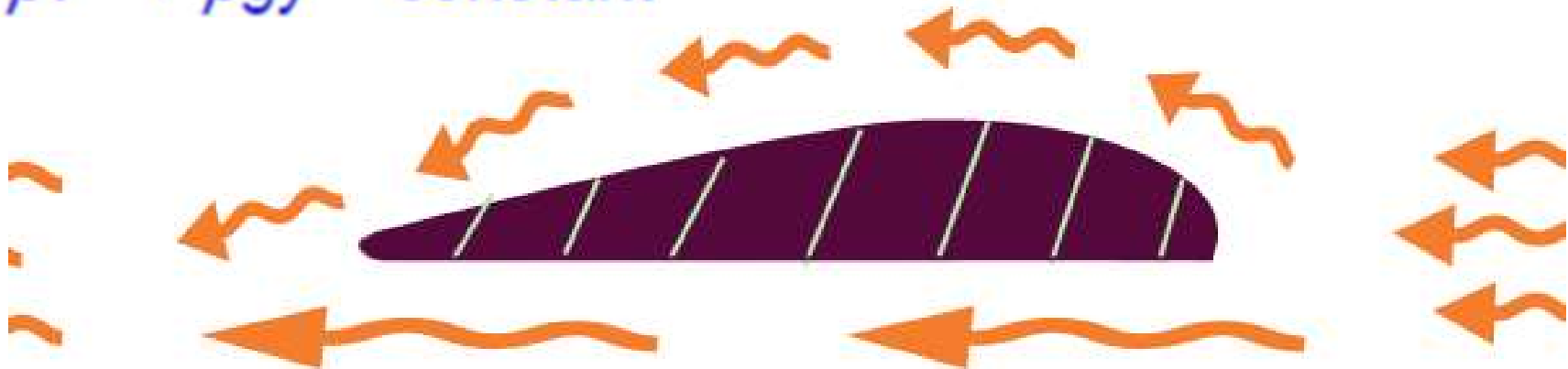
Q127

See the clip! <https://www.youtube.com/watch?v=aFO4PBolwFg>

HOW DO AIRPLANE WINGS HELP AN AIRPLANE FLY?

The top of a wing is curved.
Air moves quickly above the wing.
This causes low pressure.

$$P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}$$



The bottom of the wing is straight.
Air moves more slowly.
Pressure stays high.

Higher pressure underneath pushes the wing up
and produces lift.